**Kovalev Vyacheslav**

**Problem Set 3. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .**

1. The e-value problem for longitudinal waves in an elastic rod is:
2. Explain the physical meaning of the second boundary condition.

Boundary conditions mean that displacement at the start and the end of the rod is 0. Our rod is fixed at both ends.

1. Solve the problem analytically to find k and u.
2. Plot the eigenfunctions that correspond to the lowest three eigenvalues

Let

1. Consider elastic waves

Where

1. Find the eigenvalues numerically:

Where

Where (considering ):

Perform some transformations:

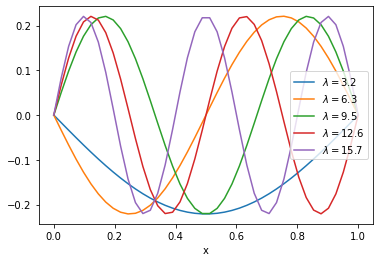
Of course, consists of elements on the main diagonal.  
This is the problem of e-values, e-vectors:

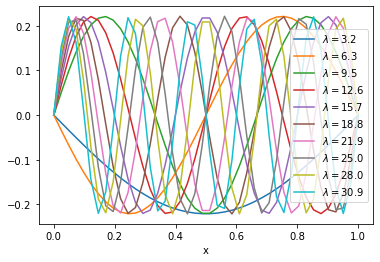
I assume (in the task it is got ), but if =>, it contradicts to condition that must be 1, because we apply rescaling. So, I will use suchand for several

|  |  |  |
| --- | --- | --- |
|  | | |
|  |  |  |
| 3.17 | 3.14 | 3.14 |
| 6.32 | 6.26 | 6.28 |
| 9.43 | 9.35 | 9.42 |
| 12.49 | 12.38 | 12.57 |
| 15.48 | 15.34 | 15.71 |

Last column corresponds to case then our equation turns to task (1) and analytical solution is

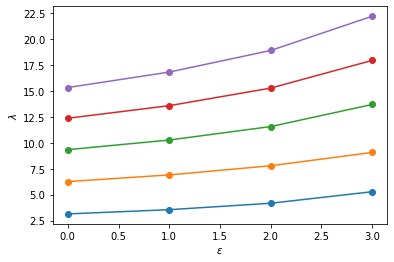
n should be chosen as large as it is possible, because it makes more accurate. I chose n = 20, because it is enough to plot by 20 points. (When is small our solution supposed to be like

1. When 



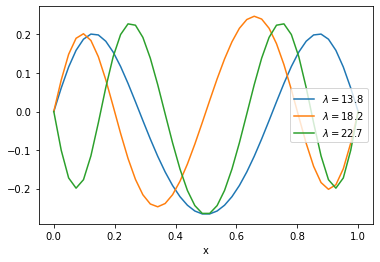
“How do they compare with those at ?”: Almost the same .

1. "Plotλ (ϵ)for the first five eigenvalues. "

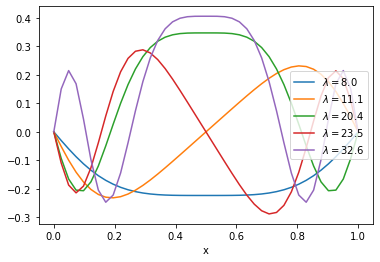


How do eigenvalues and eigenfunctions change when increases?:

For instance, as you can see it increases amplitude on the middle of . So, the more the more influence on the middle of rod.



1. What happens when becomes close to 4?



I got such eigenvectors and eigenvalues, but further increase of results in complex .

1. Find the SVD of A and illustrate on paper how transforms a vector x into Ax by a sequence of three transformations

Let’s find e-values, and e-vectors for

For :

For :

Result:

Analogically for

For ; For

In conclusion

1. Compare the result from (a) with how transforms x into .

The difference in transformation and.When and only rotate vector, and may change the norm of x too.

1. Given column vectors a and b, find the SVD of

Such as rank(A) =1

Let ; then ; =>

1. when

– it is the prev. task

1. For matrix
2. using SVD, find the orthonormal basis for the column space, C (A), and complete it with the basis for the left nullspace, to form a basis for . Write down the 4 × 4 orthogonal matrix U.

, where is orthonormal basis for C (A).

Found nonzero e-values and corresponded orthonormal e-vectors:

Basis for

Easy to show

Full U:

1. Do the same for the row space.

, where is orthonormal basis for R(A).

Found nonzero e-values and corresponded orthonormal e-vectors:

Basis for

Easy to show

Full V:

1. Write the full SVD:
2. What is the best rank-1 approximation of A?

– the closest to A, because standard deviation is smallest.